16.10.2025

MTC 04

Sem III (UG)

1 If $y = Ae^{2x} + Be^{x} + C$ ——— (1), find the diff. eqn.

Sol^n

Remember → we have to eliminate A, B and C from the differentials of y. → Very Important.

Differentiating (1), we have

$$\frac{dy}{dx} = 2Ae^{2x} + Be^{x} \quad \text{——— (2)}$$

$$\frac{d^2y}{dx^2} = 4Ae^{2x} + Be^{x} \quad \text{——— (3)}$$

$$\frac{d^3y}{dx^3} = 8Ae^{2x} + Be^{x} \quad \text{——— (4)}$$

याद रखें :
हमें किसी भी प्रकार से constants को हटाना है

Subtracting (2) from (3), we get

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} = 2Ae^{2x} \quad \text{——— (5)}$$

Subtracting (3) from (4), we get

$$\frac{d^3y}{dx^3} - \frac{d^2y}{dx^2} = 4Ae^{2x} = 2 \times 2Ae^{2x}$$

$$= 2\left(\frac{d^2y}{dx^2} - \frac{dy}{dx}\right) \quad [\text{using (5)}]$$

$$\Rightarrow \frac{d^3y}{dx^3} - 3\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = 0$$

2: If $\log y = Ax^2 + B$, find the differential eqn.

Soln: $\because \log y = Ax^2 + B$ differentiating w.r.to x, we get

$$\frac{1}{y}\frac{dy}{dx} = 2Ax$$

$$\Rightarrow \frac{dy}{dx} = 2Axy. \quad \text{---(1)}$$

Again differentiating w.r.to x, we get

$$\frac{d^2y}{dx^2} = 2A\left(x\frac{dy}{dx} + y\right) \quad \text{---(2)}$$

From (1), we have $2A = \frac{1}{xy}\frac{dy}{dx}$

Putting the value of 2A in (2), we have

$$\frac{d^2y}{dx^2} = \frac{1}{xy}\left(x\frac{dy}{dx} + y\right)\frac{dy}{dx}$$

$$\Rightarrow xy\frac{d^2y}{dx^2} = x\left(\frac{dy}{dx}\right)^2 + y\frac{dy}{dx}$$

This is the required eqn